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SESSION ID: **CRYP-W06C**

# The Key-Dependent Message Security of Key-Alternating Feistel Ciphers

**By Pooya Farshim, Louiza Khati, Yannick Seurin and Damien Vergnaud.**

ANSSI and ENS

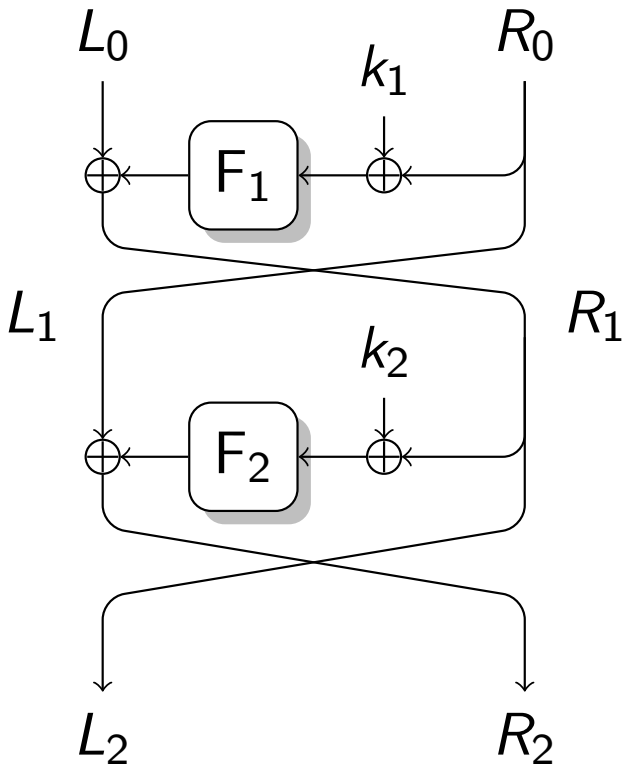
# KDM Security

- Key Dependent-Message Security
  - access to a ciphertext resulting from the encryption of **key-dependent message** [BRS03]

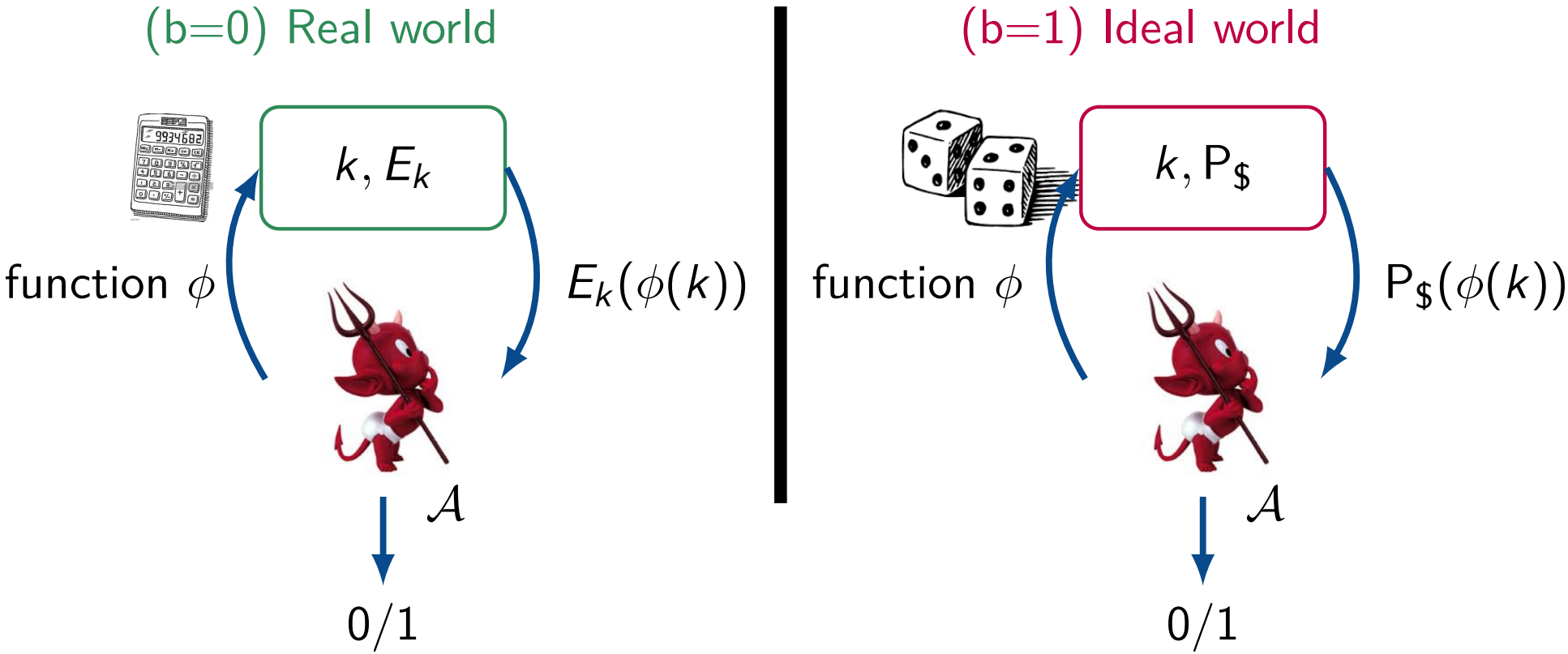
- Full disk encryption

# Key-Alternating Feistel (KAF)

- Configuration:
  - $r$ rounds
  - $r$ internal functions $\mathsf{F}_i$ ($=$ or $\neq$)
  - Key schedule:
    - $r$ keys ($=$ or $\neq$)
    - $r$ keys derivated from a master key
- Previous security analysis
  - Indistinguishability ([LS15])
  - (Indifferentiability KAF*[GL15])
- Examples:
  - DES, GOST etc.



# KDM security: Indistinguishability game




KDM-CPA adversary

$$\mathbf{Adv} = |\ \Pr[\mathcal{A} \to 1|\text{Real}] - \Pr[\mathcal{A} \to 1|\text{Ideal}]\ |$$

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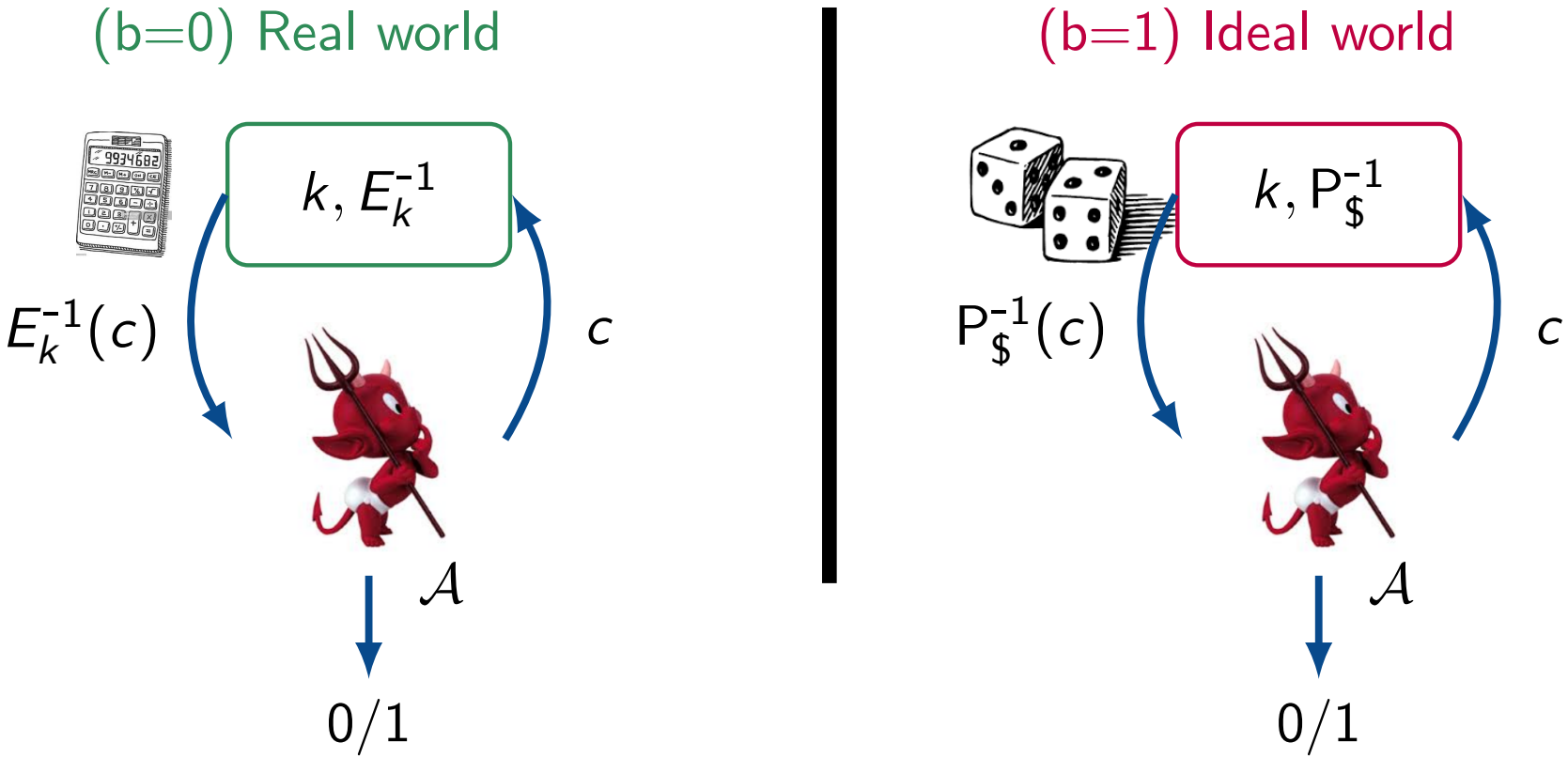



KDM-CCA adversary ("Standard" decryption)

$$\mathbf{Adv} = |\ \Pr[\mathcal{A} \to 1|\text{Real}] - \Pr[\mathcal{A} \to 1|\text{Ideal}]\ |$$

# KDM set restriction: Claw-freeness

- Find the largest set $\Phi$ of functions $\phi$ such that Adv is small
  - Including constant functions
- The KDM set $\Phi$ has to be restricted $\rightarrow$ Key extraction [FKV17]

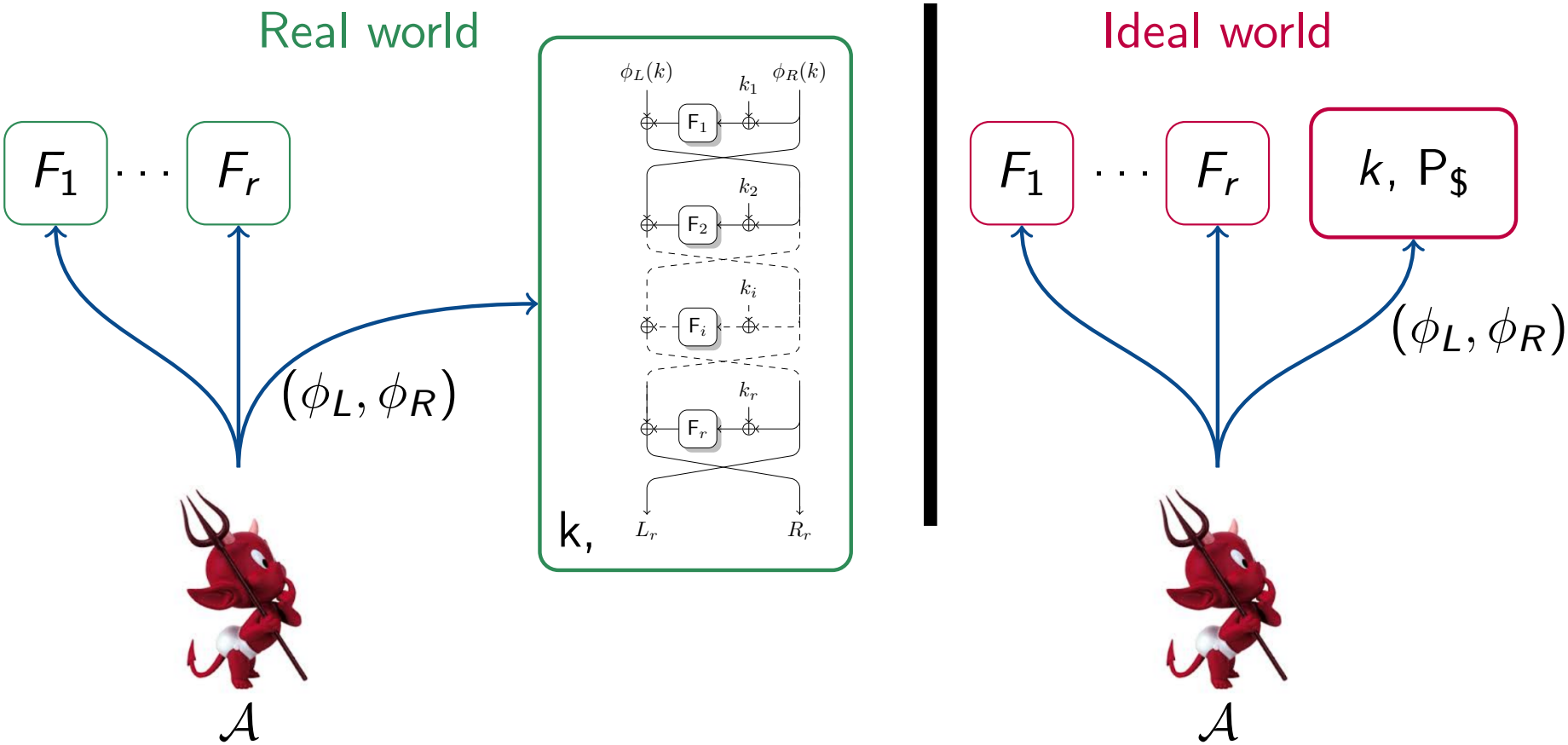
Claw-freeness of a set $\Phi$:

$$\mathbf{cf}(\Phi) := \max_{\phi_1 \neq \phi_2 \in \Phi} \Pr[k \twoheadleftarrow \mathcal{K} : \phi_1(k) = \phi_2(k)]$$

KDM security:

- Ideal-Cipher KDM-secure under claw-free sets [FKV17]
- What about Key-Alternating Feistels ?

# Securiy analysis: KDM Security game for KAF




$F_i$ uniformly random functions,
KDM set $\Phi = (\Phi_L, \Phi_R)$

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- 4-round KAF
  - Same internal function
  - **Independent keys**
  - Attack 2 queries $\rightarrow$ $\Phi_R$ offset-free

Detailed in the paper

Offset-freeness of a set $\Phi$:

$$\mathbf{of}(\Phi) := \max_{\substack{i\in\{1,..,\ell\} \\ \phi\in\Phi,\, x\in\{0,1\}^n}} \Pr[(k_1,..,k_\ell) \twoheadleftarrow \mathcal{K} : \phi(k_1,..,k_\ell) = k_i \oplus x]$$

# KDM attack

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- Sliding attack (any number of rounds)
  - Same internal function and same key
  - not CPA-secure
  - Key extraction attack (1 query)
    - $\phi_L(k) = k \oplus \mathsf{F}^2(\Delta) \oplus \Delta \oplus \mathsf{F}[\mathsf{F}(\Delta) \oplus \mathsf{F}(\mathsf{F}^2(\Delta) \oplus \Delta)$
    - $\phi_R(k) = k \oplus \mathsf{F}(\Delta) \oplus \mathsf{F}[\mathsf{F}^2(\Delta) \oplus \Delta]$
    - $R_r = k$
    - $L_r \oplus R_r = \Delta$

# Generic Proof Methodology

Ideal world (iw) Perfect world (pw) Real world (rw)

$\mathbf{Adv}_{\mathsf{iw},\mathsf{pw}}(\mathcal{A})$ $\mathbf{Adv}_{\mathsf{pw},\mathsf{rw}}(\mathcal{A})$

- Indistinguishability game with H-coefficient technique
- Triangular inequality: $\mathbf{Adv}_{\mathsf{iw},\mathsf{rw}}(\mathcal{A}) \leq \mathbf{Adv}_{\mathsf{iw},\mathsf{pw}}(\mathcal{A}) + \mathbf{Adv}_{\mathsf{pw},\mathsf{rw}}(\mathcal{A})$

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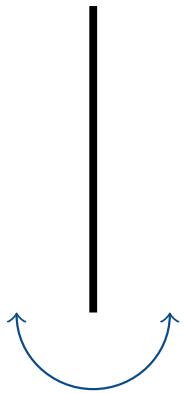

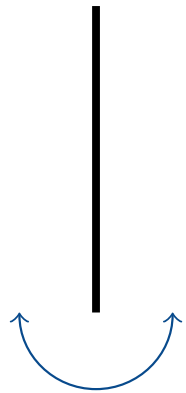

$\mathbf{Adv}_{\mathsf{iw},\mathsf{pw}}(\mathcal{A})$

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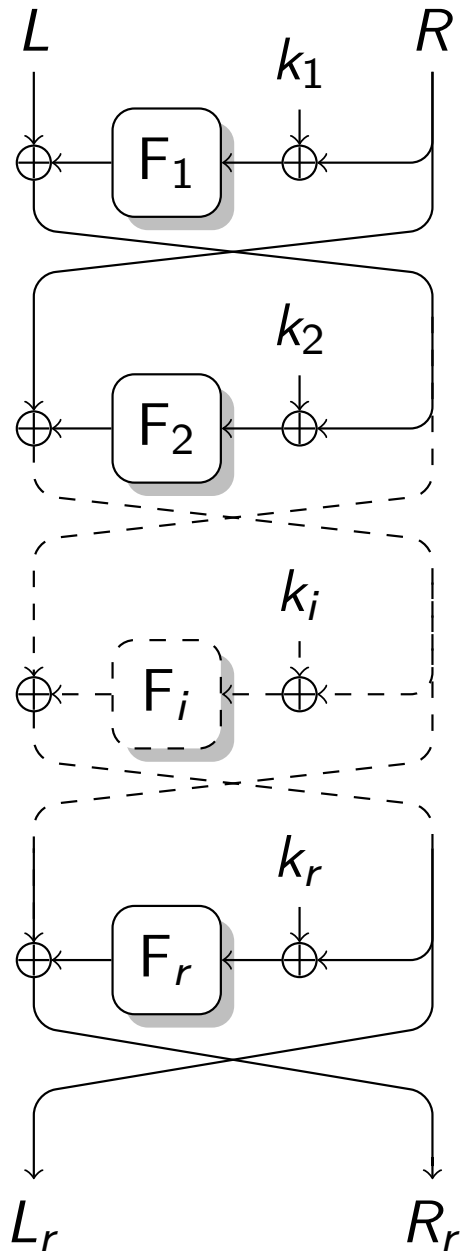
# Results: Key-Alternating Feistel

| Rounds | Functions | Keys schedule | KDM set |
|---|---|---|---|
| 4 | $\mathsf{F}_i =$ | $k_1, 0, 0, k_2$ | **cf** $\wedge$ **of** $\wedge$ **oxf** |
| 4 | $\mathsf{F}_i \neq$ | $k_i \neq$ | **cf** $\wedge$ **of**? |
| 5 | $\mathsf{F}_i =$ | $k_i \neq$ | **cf** $\wedge$ **of**? |
| ? | $\mathsf{F}_i =$ | $k_i \neq$ | **cf** |



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Xor-offset-freeness of a set $\Phi$:

$$\mathbf{oxf}(\Phi) := \max_{\substack{i \neq j \in \{1,..,\ell\} \\ \phi \in \Phi,\, x \in \{0,1\}^n}} \Pr[(k_1, .., k_\ell) \twoheadleftarrow \mathcal{K} : \phi(k_1, .., k_\ell) = k_i \oplus k_j \oplus x]$$

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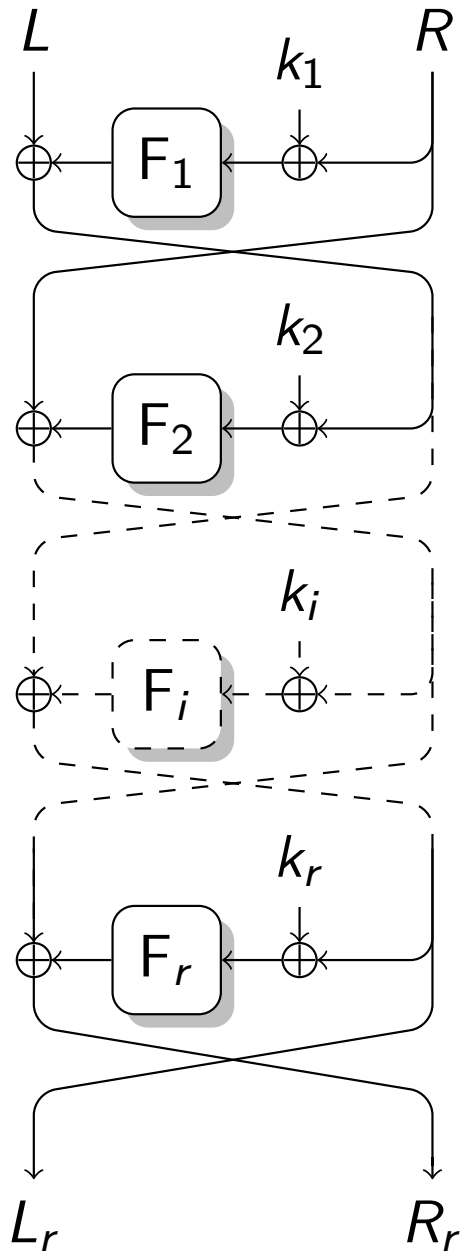

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# Thank you for your attention.

## Questions ?


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